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CA18232 – MATHEMATICAL MODELS FOR INTERACTING DYNAMIC NETWORKS - PART 2 ACOUSTIC CLOAKS

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Abstract The intention of this work is to discuss some mathematical and computational models used to integrate and interpret heterogeneous engineering data, understanding fundamental principles of dynamics in the frame of the action CA18232 – Mathematical models for interacting dynamic networks. Thus, such tools are now routinely used in the theoretical and experimental systematic investigation of dynamical systems. The idea of invisibility (objects or waves) has fascinated people for years and has been an inspiration for myths, novels and films, from the Greek legend of Perseus versus Medusa to Well's Invisible Man. Newton had viewed the colour as a physical problem, involving light striking objects and entering our eyes. For Goethe (1810) the sensations of colour reaching our brain are also shaped by our perception, the mechanics of human vision and the way our brains process information.

Key words: CA18232, dynamics networks, acoustic cloaks.

1. INTRODUCTION

Many physical, biological, chemical, financial or even social phenomena can be described by dynamical systems. It is quite common that the dynamics arises as a compound effect of the interaction between sub-systems in which case we speak about coupled systems. This Action shall study such interactions in particular cases from three points of view: the abstract approach to the theory behind these systems, applications of the abstract theory to coupled structures like networks, neighbouring domains divided by permeable membranes, possibly non-homogeneous simplicial complexes, etc., modelling real-life situations within this framework.

The purpose of this Action is to bring together leading groups in Europe working on a range of issues connected with modelling and analysing the mathematical models for dynamical systems on networks. It aims to develop a semigroup approach to various (non-)linear dynamical systems on networks as well as numerical methods based on modern variational methods and applying them to road traffic, biological systems, and further real-life models. The Action also explores the possibility of estimating solutions and long-time behaviour of these systems by collecting basic combinatorial information about underlying networks.

The intention of this work is to discuss the transformation acoustics which opens a new avenue towards the design of acoustic metamaterials, which are materials engineered at the subwavelength scale in order to mimic the parameters in wave equations. The design of the acoustic cloaking is based on the property of equations to be invariant under a coordinate transformation, i.e. a specific spatial compression is equivalent to a variation of the material parameters in the original space. In this chapter, the sound invisibility performance is discussed for spherical cloaks. The original domain consists of an alternation of concentric layers made from piezoelectric ceramics and epoxy resin, following a triadic Cantor sequence. The spatial compression, obtained by applying the concave-down transformation, leads to an equivalent domain with an inhomogeneous and anisotropic distribution of the material parameters.

Also, the transformation acoustics makes possible the architecture, modeling and simulation of a new class of sonic composites with scatterers made of various materials and having various shapes embedded in an epoxy matrix. The design of acoustic scatterers is based on the property of Helmoltz equations to be invariant under a coordinate transformation, i.e. a specific spatial compression is equivalent to a new material in a new space. In this chapter, the noise suppression for a wide full band-gap of frequencies is discussed for spherical shell scatterers made of auxetic materials (materials with negative Poisson’s ratio). The original domain consists of spheres made from conventional foams with positive Poisson’s ratio. The spatial compression is controlled by the coordinate transformation, and leads to an equivalent domain filled with an auxetic material. The coordinate transformation is strongly supported by the manufacturing of auxetics which is based on the pore size reduction through radial compression molds.

2. 3D-DIMENSIONAL SPHERICAL ACOUSTIC CLOAKING

Dynamic network analysis is an emergent scientific field that brings together problems related to the field of nonlinear dynamical systems. These problems request strong knowledge related to applied mathematics and mechanics, since vibrations of real engineering systems are dynamic phenomena described by differential or partial differential equations which are nonlinear and often strongly nonlinear.

There are two aspects of this field. The first is the solving the nonlinear dynamical equations and the dynamic analysis of data. The second is the utilization of simulation to address issues of network dynamics. An early study of the dynamics of link utilization in very large-scale complex networks provides evidence of dynamic centrality, dynamic motifs, stability and chaos in structural interactions

This paper introduces the fundamental ideas underlying some mathematical methods in the frame of the action CA18232 – Mathematical models for interacting dynamic networks.

The solitons are localized waves that conserve their properties even after interaction among them, and then act somewhat like particles. The equations which describe the solitons have interesting properties: an infinite number of local conserved quantities, an infinite number of exact solutions expressed in terms of the Jacobi elliptic functions or the hyperbolic functions and the simple formulae for nonlinear superposition of explicit solutions. Such equations were considered integrable or more accurately, exactly solvable.

Transformation acoustics is the key for the design of acoustic metamaterials, which are materials engineered at the subwavelength scale [1-3]. Recent works show that acoustic metamaterials could cloak regions of space, making them invisible to sound [5-7]. We refer to acoustic cloaking which occurs when a medium contains a region in which noisy objects can be acoustically hidden. It is easy to imagine an object invisible to sound by building a box around it to prevent the wave from reaching the object.

The principle how to cloak a region of space to make its contents invisible or transparent to waves was discussed by Miller [7]. Recent papers [8-13] have used the coordinate invariance of Maxwell's equations to show how a region of the space can be made inaccessible to electromagnetic waves by surrounding it with a suitable dielectric shield. Kohn et al. 2008 have analyzed the shortcomings of the aforementioned papers: (a) the cloaks they consider are rather singular; and (b) the analysis by

Greenleaf, Lassas and Uhlmann [14] does not apply in the space dimension n = 2, and provided a treatment that remedies these shortcomings. They have shown how a regular near-cloak can be obtained using a nonsingular change of variables, and proved that the change-of-variable-based scheme achieves perfect cloaking in any dimension $n \geq 2$.

As an alternative to a box made from a metamaterial, sonic composites (or sonic crystals) exhibit the full band-gaps, where the sound is not allowed to propagate due to complete reflections [22, 23]. Cummer *et al*. [15] derived the mass density and bulk modulus of a spherical shell that can eliminate scattering from an arbitrary object in the interior of the acoustic shell. Calculations confirmed that the pressure and velocity fields were smoothly bent and excluded from the central region as for previously reported electromagnetic cloaking shells. It is also interesting to note that the ideal 3D acoustic cloaking parameters are similar in structure to the 2D electromagnetic and 2D acoustic parameters in that they contain singularities on the interior edge of the cloak. Cummer and Schurig [16] have demonstrated that in a 2D geometry, the acoustic equations in a fluid are identical in form to the single polarization Maxwell equations via a variable exchange that also preserves boundary conditions.

In this chapter, we apply the 3D concave-down transformation to design a spherical cloak which surrounds a noisy machine, see Fig.1. The original domain is a sphere of radius R_2, consisting of an alternation of concentric layers made from piezoelectric ceramics and epoxy resin, following a triadic Cantor sequence. After the transformation, the cloak contains a region $r < R_1$ which is filled with air and contains the noisy source, while the shell $R_1 < r < R_2$ is filled by the nonlinearly transformed material.

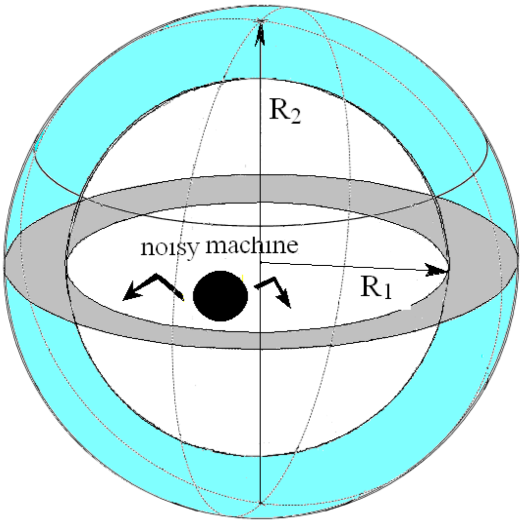

Fig. 1. Sketch of the spherical cloak surrounding a noisy machine.

A finite size object surrounded by a coating consisting of a specially designed metamaterial would become invisible for electromagnetic waves at any frequency [5]. In acoustics, the idea of the invisibility cloak is that the sound sees the space differently [17]. For the sound, the concept of distance is modified by the acoustic properties of the regions through which the sound travels. In geometrical acoustics, we are used to the idea of the acoustical path; when travelling an infinitesimal distance $\mathrm{d}s$, the corresponding acoustical path length is $c^{-1}\mathrm{d}s$, where $c^{-1} = \sqrt{\rho / \kappa}$ with ρ is the fluid density and κ is the compression modulus of the fluid [23, 30].

The 3D acoustic equation for the pressure waves propagating in a bounded fluid region $\Omega \subset \mathrm{R}^3$ is

$$\nabla \cdot (\underline{\underline{\rho}}^{-1} \nabla p) + \frac{\omega^2}{\kappa} p = 0 , \tag{1}$$

where p is the pressure, $\underline{\underline{\rho}}$ is the rank-2 tensor of the fluid density, κ is the compression modulus of the fluid, and ω is the wave frequency.

Let us consider the geometric transformation from the coordinate system (x', y', z') of the compressed space to the original coordinate system (x, y, z), given by $x(x', y', z')$, $y(x', y', z')$ and $z(x', y', z')$. The change of coordinates is characterized by the transformation of the differentials through the Jacobian $J_{xx'}$ of this transformation, i.e.

$$\begin{pmatrix} \mathrm{d}x \\ \mathrm{d}y \\ \mathrm{d}z \end{pmatrix} = J_{xx'} \begin{pmatrix} \mathrm{d}x' \\ \mathrm{d}y' \\ \mathrm{d}z' \end{pmatrix}, \quad J_{xx'} = \frac{\partial(x, y, z)}{\partial(x', y', z')}. \tag{2}$$

From the geometrical point of view, the change of coordinates implies that, in the transformed region, one can work with an associated metric tensor [4, 18]

$$T = \frac{J_{xx'}^{\mathrm{T}} J_{xx'}}{\det(J_{xx'})}. \tag{3}$$

In terms of the acoustic parameters, one can replace the material from the original domain (homogeneous and isotropic) by an equivalent compressed one that is inhomogeneous (its characteristics depend on the spherical (r', θ', ϕ') coordinates) and anisotropic (described by a tensor), and whose properties, in terms of $J_{x'x}$, are given by

$$\underline{\underline{\rho}}' = J_{x'x}^{-\mathrm{T}} \cdot \rho \cdot J_{x'x}^{-1} \cdot \det(J_{x'x}), \quad \kappa' = \kappa \det(J_{x'x}), \tag{4}$$

or, equivalently, in terms of $J_{xx'}$

$$\underline{\underline{\rho}}' = \frac{J_{xx'}^{\mathrm{T}} \cdot \rho \cdot J_{x'x}}{\det(J_{xx'})}, \quad \kappa' = \frac{\kappa}{\det(J_{xx'})}. \tag{5}$$

Here, $\underline{\underline{\rho}}'$ is a second order tensor. When the Jacobian matrix is diagonal, (4) and (5) can be more easily written. Multiplying (1) by a test function φ and integrating by parts, one obtains

$$-\int_{\Omega} \left(\nabla_{(x,y,z)} \varphi \cdot \underline{\underline{\rho}}^{-1} \nabla_{(x,y,z)} p \right) \mathrm{d}V + \int \left(\omega^2 \kappa^{-1} p \varphi \right) \mathrm{d}V = 0. \tag{6}$$

In (6) the surface integral, corresponding to a Neumann integral over the boundary $\partial\Omega$, is zero. By applying the coordinate transformation $(x, y, z) \to (x', y', z')$ to (6) and using (2), one obtains

$$-\int_{\Omega} \left(J_{x'x}^{\mathrm{T}} \nabla_{(x',y',z')} \varphi \cdot \underline{\underline{\rho}}^{-1} J_{x'x}^{\mathrm{T}} \nabla_{(x,y,z)} p \right) \det(J_{xx'}) \mathrm{d}V' + \int \left(\det(J_{xx'}) \omega^2 \kappa^{-1} p \varphi \right) \mathrm{d}V' = 0, \tag{7}$$

in terms of $J_{xx'}$, and

$$-\int_{\Omega} \left(\left(\nabla_{(x',y',z')} \varphi \right)^{\mathrm{T}} \frac{J_{x'x} \underline{\underline{\rho}}^{-1} J_{x'x}^{\mathrm{T}}}{\det(J_{x'x})} \nabla_{(x',y',z')} p \right) \mathrm{d}V' + \int \left(\frac{\kappa^{-1}}{\det(J_{x'x})} \omega^2 p \varphi \right) \mathrm{d}V' = 0, \tag{8}$$

in terms of $J_{x'x}$.

The geometric transformation may be linear or nonlinear. Qiu *et al.* [19] classified the geometric transformation functions in terms of the negative (i.e., concave-down) or positive (i.e., concave-up) sign of the second order derivative of this function. All transformations, i.e. linear, concave-up and concave-down transformations, are perfect cloaks for the exact inhomogeneous design.

The concave-down nonlinear transformation compresses a sphere of radius R_2 in the original space Ω into a shell region $R_1 < r' < R_2$ in the compressed space Ω' as

$$r(\beta) = \frac{R_2^{\beta+1}}{R_2^{\beta} - R_1^{\beta}}\left(1 - \left(\frac{R_1}{r'}\right)^{\beta}\right), \tag{9}$$

where β denotes the degree of the nonlinearity in the transformation. By taking $\beta \to 0$ in (3.9), the linear case is obtained, namely

$$r(\beta) = \frac{R_2 \mathrm{Ln}(r'/R_1)}{\mathrm{Ln}(R_2/R_1)}. \tag{10}$$

All curves belonging to (9) have negative second order derivative with respect to the physical space r'. This class of transformations is termed as the *concave-down* transformation. The transformation function (9) depends on the radial component r' in the spherical coordinate system (r', θ', ϕ').

The concave-up nonlinear transformation compresses a sphere of the radius R_2 in the original space Ω into a shell region $R_1 < r' < R_2$ in the compressed space Ω' as

$$r(\beta) = \frac{R_2 R_1^{\beta}}{R_2^{\beta} - R_1^{\beta}}\left(\left(\frac{r'}{R_1}\right)^{\beta} - 1\right). \tag{11}$$

As $\beta \to 0$, one obtains again the linear case (10). This class of transformations is termed as the *concave-up* transformation because (11) has positive second order derivatives.

All curves belonging to (9) have negative second order derivative with respect to the physical space r'. This class of transformations is termed as the *concave-down* transformation. The nonlinear transformation function in (9) only depends on the radial component r' in the spherical coordinate system (r', θ', ϕ'). The cloak properties in the both transformed coordinates are given by (4) and (5) where $J_{r'r} = \partial r'/\partial r$.

Milton *et al.* [1] showed that geometric transformations cannot be applied to equations which are not invariant under coordinate transformations and, consequently, if cloaking exists for such equations (for example the elasticity equations), it would be of a different nature from acoustic and electromagnetic. The existence of an acoustic cloaking indicates that cloaks might possibly be built for other wave systems, including seismic waves that travel through the earth and the waves at the surface of the ocean (Cummer *et al.* [16]).

Farhat *et al.* [21] discussed a special case of thin-elastic plates, for which the elasticity tensor can be represented in a cylindrical basis by a diagonal matrix with two (spatially varying) non-vanishing entries. Indeed, the equations governing the propagation of elastodynamic waves with a time harmonic dependence are written, in a weak sense, as

$$\nabla \cdot C : \nabla u + \rho\omega^2 u = 0, \tag{12}$$

where ρ is the scalar density of an isotropic heterogeneous elastic medium, C is the fourth-order elasticity tensor, ω is the wave angular frequency, and $u(x_1, x_2, x_3, t) = u(x_1, x_2, x_3)\exp(-\mathrm{i}\omega t)$ is the vector displacement. Under a change of coordinates (x', y', z') to (x, y, z) such that $u'(x') = J_{x'x}^{-\mathrm{T}} u(x)$, $J_{x'x} = \dfrac{\partial(x', y', z')}{\partial(x, y, z)}$, Eq. (12) takes the form

$$\nabla''\cdot(C'+S'):\nabla' u' + \underline{\underline{\rho}}'\omega^2 u' = D':\nabla' u' \quad , \tag{13}$$

which preserves the symmetry of the new elasticity tensor $C'+S'$. Equation (13) contains two third-order symmetric tensors S' and D' with $D'_{pqr} = S'_{qrp}$, and a second-order tensor ρ'_{pq}.

Our intention is to replace a material made from concentric homogeneous and isotropic layers situated in the original spherical domain by an equivalent compressed inhomogeneous anisotropic material described by the transformation matrix. These kinds of materials are not naturally occurring. However, recent advances in metamaterials are encouraging for such an approach to constitutive parameters required for cloaking. Metamaterials are materials with subwavelength microstructures that are designed to have desired physical and acoustical properties. Despite the latest advances in metamaterials, we do not currently have the ability to manufacture a cloak with ideal constitutive parameters..

Let us suppose that the original domain Ω is a sphere of radius R_2. The sphere consists of an alternation of concentric layers made from piezoelectric ceramics and epoxy resin, following a triadic Cantor sequence up to the fourth generation (31 elements). A sketch of this material is represented in Fig. 2 [8]. The dashed regions are occupied by piezoelectric ceramics of total volume V^p and boundary external surface S_1^p. The white regions are occupied by epoxy-resins of total volume V^e and boundary external surface S_1^e . The lateral surfaces are S_2 , while the interfaces between constituents are denoted by I^{pe}. Let the regions occupied by the plate be $\Omega = V^p \cup V^e$, where V^p and V^e are the regions occupied by the PZ and ER layers, respectively.

The governing equations of this composite are

$$\rho \ddot{u}_i = t_{ij,j}\,, \text{ in } \Omega\,, \tag{14}$$

$$D_{i,i} = 0\,, \quad E_i + \varphi_{e,i} = 0\,, \text{ in } V^p\,, \tag{15}$$

$$t_{ij} = \lambda^p \varepsilon_{kk}\delta_{ij} + 2\mu^p \varepsilon_{ij} - e_k^p E_k \delta_{ij}\,, \quad \text{in} V^p\,, \tag{16}$$

$$t_{ij} = \lambda^e \varepsilon_{kk}\delta_{ij} + 2\mu^e \varepsilon_{ij}\,, \quad \text{in } V^e\,, \tag{17}$$

$$D_i = \overline{\varepsilon}^p E_i - e_i^p \varepsilon_{kk}\,, \quad \text{in } V^p\,, \tag{18}$$

$$\varepsilon_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i})\,, \quad \text{in } \Omega\,. \tag{19}$$

Here, indices p and e denote the piezoelectric (PZ) and non-piezoelectric (ER) materials, respectively, ρ is the density, u_i, $i=1,2,3$, are the components of the displacement vector, t_{ij}, $i=j=1,2,3$, are the components of the stress tensor, D_i, $i=1,2,3$, are the components of the electric induction vector, E_i, $i=1,2,3$, are the components of the electric field and φ_e is the electric potential, ε_{ij}, $i=j=1,2,3$, are the components of the strain tensor, λ, μ are the Lamé constants, $\overline{\varepsilon}^p$ is the dielectric constant and e_i^p ($e_3^p = e_2^p = e_1^p$) are the piezoelectricity coefficients. The coordinate x_1 is directed along the radial direction, x_3 is directed along the circumferential direction, while x_2 is located within the layer.

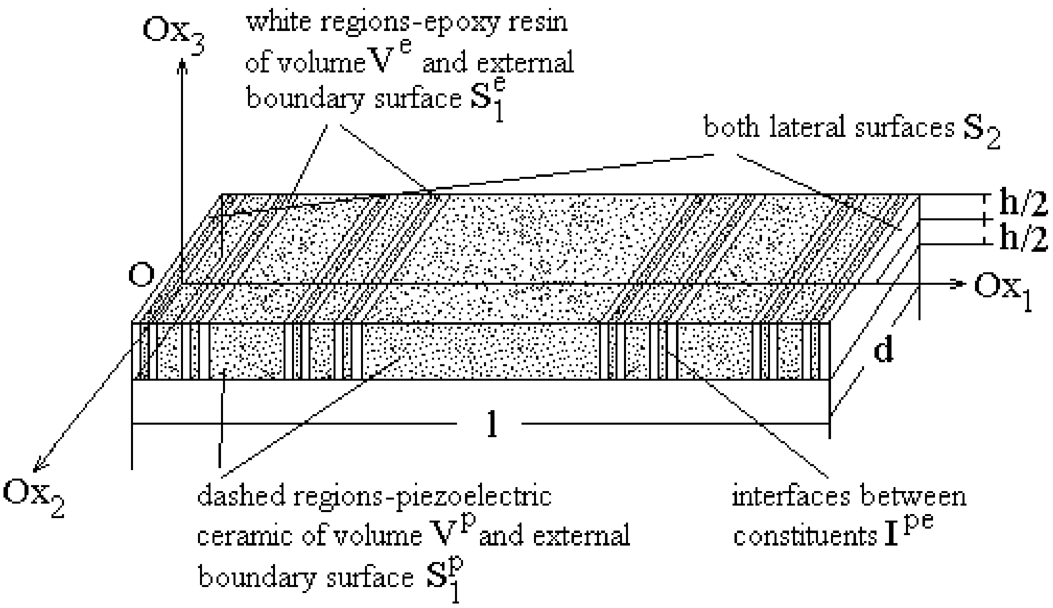

Fig. 2. The Cantor-like structure.

The scalar elastic potential φ, and the components ψ_1, ψ_2, ψ_3 of the vectorial elastic potential, defined as

$$u_1 = \varphi_{,1} - \psi_{2,3}, \quad u_2 = \psi_{1,3} - \psi_{3,1}, \quad u_3 = \varphi_{,3} + \psi_{2,1}, \tag{20}$$

and the electric potential φ_e, are expressed using the theta-function

$$\varphi(x_1, x_2, x_3, t) == \varphi_0(t)\Delta(\log\Theta(x_1, x_2, x_3)),$$

$$\psi_i(x_1, x_2, x_3) = \psi_{i0}(t)\Delta(\log\Theta(x_1, x_2, x_3)), \ i = 1, 2, 3,$$

$$\varphi_e(x_1, x_2, x_3) = \varphi_{e0}(t)\Delta(\log\Theta(x_1, x_2, x_3)). \tag{21}$$

The theta function Θ is the solution of the von Karman equation

$$\nabla \cdot \zeta_{p,e}^{-1}\nabla(\Delta\nabla \cdot \zeta_{p,e}^{-1}\nabla\Theta) - \Lambda^{-1}\gamma_0^4\Theta = 0, \tag{22}$$

where $\zeta = E^{-1/2}$, E is the effective Young modulus of the composite, $\gamma_0^4 = \omega^2\rho h / D_0$, D_0 is the flexural rigidity of the plate, ρ its effective density, h its thickness, $\Lambda = \rho^{-1}$ and ω the frequency. Eq. (22) can be factorized as a Helmholtz operator and an anti-Helmholtz operator (i.e. with an opposite sign for the spectral parameter)

$$(\nabla^2 + \gamma_0^2)((\nabla^2 - \gamma_0^2)\Theta = 0, \tag{23}$$

where for simplicity we have taken $\zeta = \Lambda = 1$. We write the Helmholtz equation in the coordinate system (x_1, x_2, x_3) as

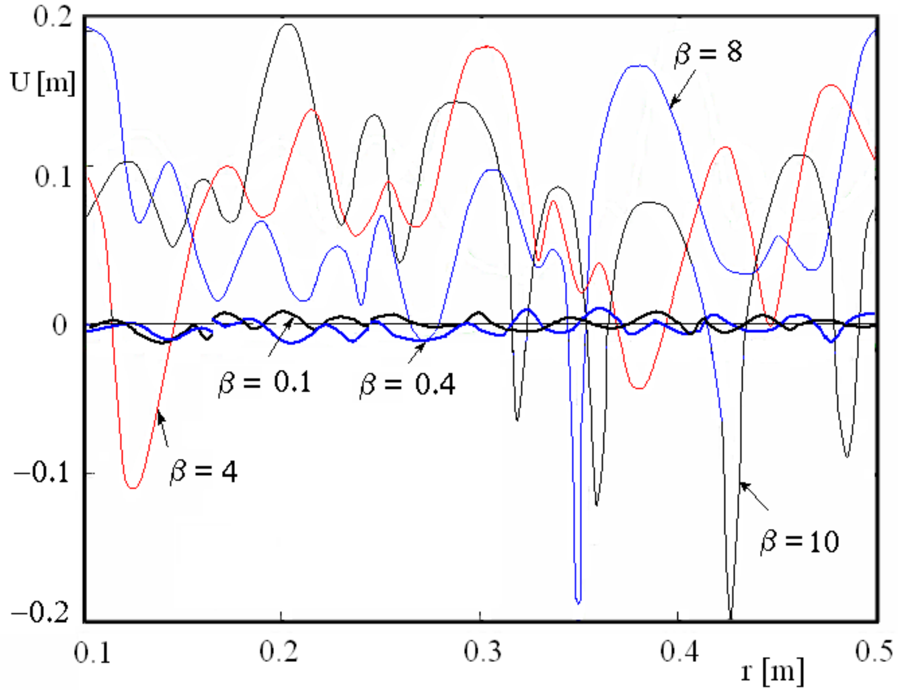

Fig. 3. Variation of the displacement amplitude with respect to β in the region $r \le R_1$.

$$\nabla\cdot(\zeta^{-1}\nabla\Theta) + \omega^2\Lambda^{-1}\Theta = 0\,. \qquad (24)$$

Let us apply the concave-down transformation (9) to (22), which compresses the original domain Ω occupied by a sphere of radius R_2 into a shell region $R_1 < r' < R_2$ in the compressed space Ω' characterized by

$$\underline{\underline{\zeta}}'^{-1}_{p,e}(r') = J^{\mathrm{T}}_{rr'}\zeta^{-1}_{p,r}(r)J_{rr'}/\det(J_{rr'})\,,\ \ \underline{\underline{\Lambda}}'^{-1}(r') = J^{\mathrm{T}}_{rr'}\Lambda^{-1}(r)J_{rr'}/\det(J_{rr'})\,,\ \ J_{rr'} = \partial r/\partial r'\,, \qquad (25)$$

In the new coordinates, the transformed equation (22) now reads as

$$\nabla\cdot\underline{\underline{\zeta}}^{-1}_{p,e}\nabla(\Delta_{33}\nabla\cdot\underline{\underline{\zeta}}^{-1}_{p,e}\nabla\Theta') - \Lambda^{-1}_{33}\gamma_0^4\Theta' = 0\,, \qquad (26)$$

where $\underline{\underline{\zeta}}^{-1}_{p,e}$ is the upper diagonal part of the inverse of $\underline{\underline{\zeta}}$ and Λ^{-1}_{33} is the third diagonal entry of $\underline{\underline{\Lambda}}^{-1}$

The cloak has the inner radius $R_1 = 0.5\text{m}$ and outer radius $R_2 = 1\text{m}$. The concave-down transformation presents an overlapping for all mapping curves for $\beta < 0.1$, which means the same results in applications. The effect of β on the amplitude of displacements, which vary from $-U$ to U ($U = \sqrt{u_1^2 + u_2^2 + u_3^2}$) inside the cloak $r \le R_1$, is illustrated in Fig. 3.

It can be seen that when β increases, the amplitude increases significantly inside the region $r \le R_1$ of the cloak. This is due to the fact that more energy is guided towards the inner boundary $r = R_1$, which in turn makes the cloaked object more *acoustically visible* to external incidences. For $\beta = 0.1$ and 0.4, the acoustically invisibility is good. The effect of β on the amplitude of displacements in the shell region $R_1 < r < R_2$ is illustrated in Fig. 4. In a similar manner, when β increases, the amplitude increases significantly in the shell region of the cloak.

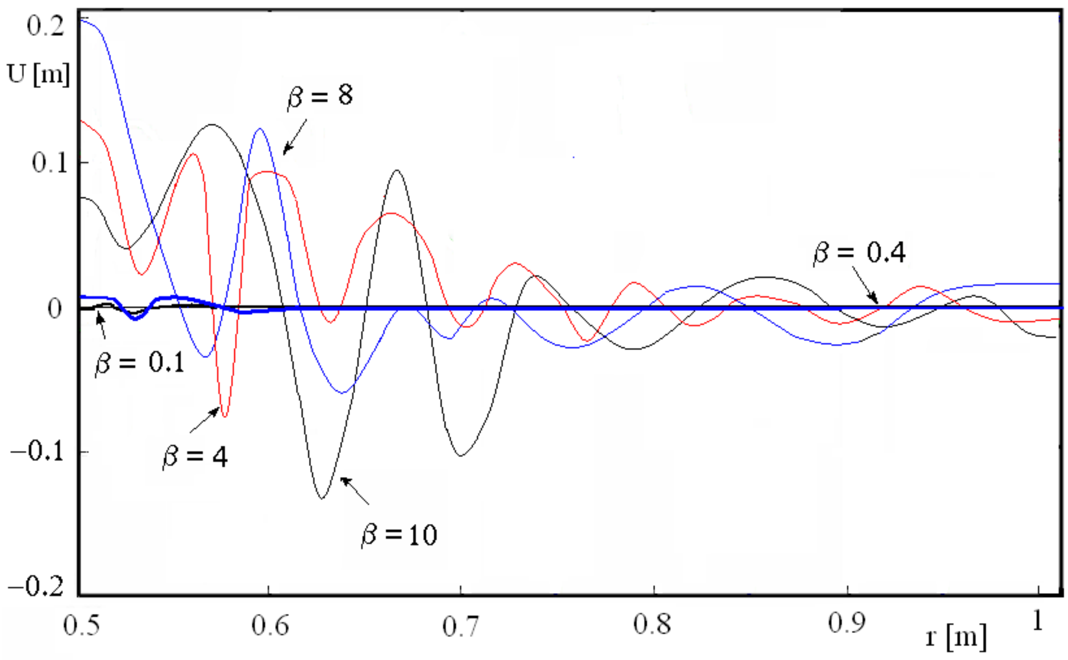

Fig. 4. Variation of the displacement amplitude with respect to β in the region $R_1 < r < R_2$.

The absence of the scattering of waves generated by an external source outside the cloak is observed in Fig. 5 for $\beta = 0.1$. The waves are smoothly bent around the central region inside the cloak. The results reported in Fig.5 show that the wave field inside the cloak, i.e. the inner region of radius R_1 which surrounds the noisy machine, is completely isolated from the region situated outside the cloak. The waves generated by a noisy source are smoothly confined inside the inner region of the cloak, and the sound invisibility detected from the observer is proportional to β. The inner region is acoustically isolated and the sound is not detectable by an exterior observer because the amplitudes on the boundary vanish.

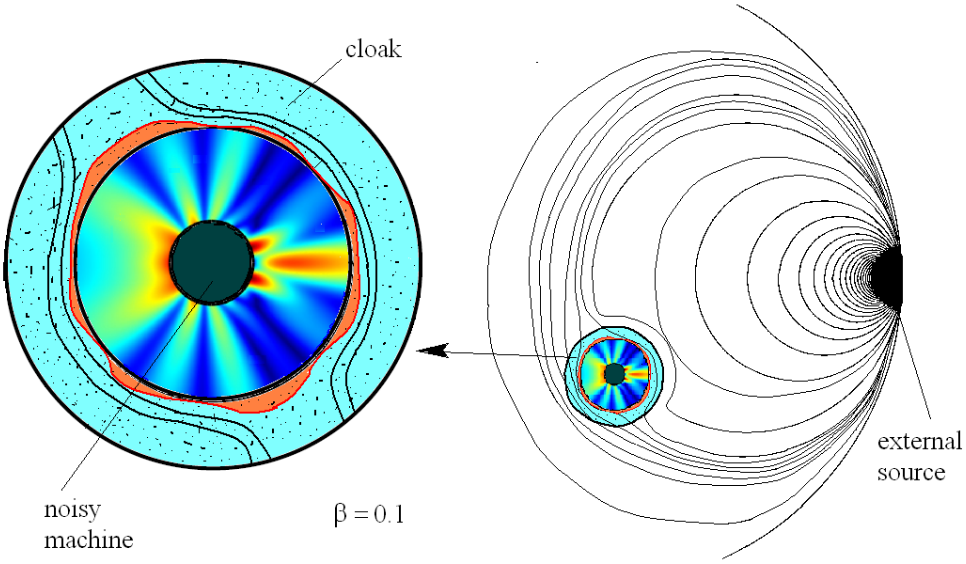

Fig. 5. The wave fields inside and outside the cloak for $\beta = 0.1$.

The domain $r < R_1$ is an acoustic invisible domain for exterior observers. The waves generated by the exterior source outside the cloak do not interact with the interior field of waves. A possible interaction or coupling between the internal and external wave fields is cancelled out by the presence of the shell region $R_1 < r < R_2$ filled with metamaterial

Hence, we can conclude that for the concave-down spherical cloaks, smaller values for β lead to a smaller disturbance in the acoustic fields in both the inner and the outer spaces $r < R_2$ and $r > R_2$, respectively.

3. CONCLUSIONS

We realise our review of cloaking results is far from being exhaustive, and it moreover remains to explore many other aspects such as connections between focussing effects and cloaking within negatively refracting media. Recent progress in the understanding of the physics of surface waves propagating on structured meta-surfaces opens unprecedented avenues the idea of invisibility (objects or waves).

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