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DAMPING OF ELASTOMERIC ANTI-SEISMIC AND ANTI-VIBRATION SYSTEMS

Polidor BRATU
University "Dunărea de Jos", Galaţi
General Manager, ICECON S.A., Bucharest

Corresponding author: Polidor BRATU, E-mail: icecon@icecon

Abstract Owing to the large variety of the existing elastomers based on natural rubber and chemical additives improving the assembly elasticity or plasticity, the dynamic analysis becomes a well-known method intended for testing and qualification of the elasticity as well as the damping. This paper presents two cases regarding elastomer systems behaving according to the linear viscoelastic law or the hysteretic law, being independent in respect to the excitation frequency.

Key words: Elastomer systems, hysteretic law.

1. VIBRATION DAMPING CHARACTERISTIC

For a system with several degrees of freedom with M the inertia matrix, the discrete viscoelastic bonds can be expressed both by the elasticity matrix K and by the attenuation matrix C. Based on the Rayley model, the linear combination is adopted in the shape of

$$\mathbf{C} = \alpha\mathbf{M} + \beta\mathbf{K} \tag{1}$$

where α and β are real and positive constants.

Performing the modal transposition by multiplying to the right by Φ and to the left by Φ^T the relation (1), where Φ is the matrix of the modal vectors, we have

$$\Phi^T\mathbf{C}\Phi = \alpha\Phi^T\mathbf{M}\Phi + \beta\Phi^T\mathbf{K}\Phi \tag{2}$$

in which $\mathbf{C}_0, \mathbf{M}_0$ and $\mathbf{K}_0$ are the diagonal matrices of the matrices $\mathbf{C}, \mathbf{M}$ and respectively $\mathbf{K}$, as a result of the process of modal transformation, i.e. diagonalization. Thus, we have

$$\mathbf{C}_0 = \alpha\mathbf{M}_0 + \beta\mathbf{K}_0 \tag{3}$$

Elements of matrices $\mathbf{C}_0, \mathbf{M}_0$ and $\mathbf{K}_0$, located diagonally, are in the shape of

$$\mathbf{C}_{0i} = \Phi_i^T\mathbf{C}\Phi_i$$
$$\mathbf{M}_{0i} = \Phi_i^T\mathbf{M}\Phi_i$$
$$\mathbf{K}_{0i} = \Phi_i^T\mathbf{K}\Phi_i$$

Depending on the matrix $\mathbf{K}_0 = \mathbf{M}_0\Omega_0^2$, $\Omega_0^2 = I\omega^2 = \mathrm{diag}\,[\omega_i^2]$, for the module i we have

$$\Phi_i^T\mathbf{C}\Phi_i = \alpha\Phi_i^T\mathbf{M}\Phi_i + \beta\Phi_i^T\mathbf{K}\Phi_i$$

where from

$$\mathbf{C}_{0i} = \alpha\mathbf{M}_{0i} + \beta\mathbf{K}_{0i}$$

or

$$2\zeta_i\omega_i\mathbf{M}_{0i} = \alpha\mathbf{M}_{0i} + \beta\mathbf{M}_{0i}\Omega_{ni}^2$$

Thus, we obtain

$$2\zeta_i\omega_i = \alpha + \beta\omega_i^2 \tag{4}$$

For module j, similarly, we can write

$$2\zeta_j\omega_j = \alpha + \beta\omega_i^2 \tag{5}$$

and from the equations (4) and (5) results

$$\alpha = 2\omega_i\omega_j\frac{\zeta_i\omega_i - \zeta_j\omega_j}{\omega_j^2 - \omega_i^2}$$
$$\beta = 2\frac{\zeta_j\omega_j - \zeta_i\omega_i}{\omega_j^2 - \omega_i^2} \qquad (6),(7)$$

The module r, cu $r = 1,2, \dots, i, \dots j, \dots, n$, can be written as

$$\zeta_r = \frac{\alpha}{2\omega_r} + \frac{\beta}{2}\omega_r \tag{8}$$

which is represented graphically in Figure 1. We see that it can highlights three significant types of damping properties.

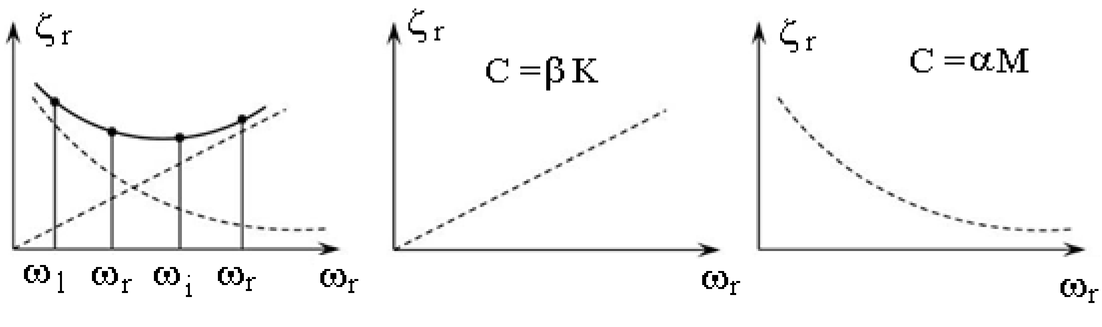

Fig. 1. The definition of the module *r.*

2. DAMPING PROPORTIONAL TO STIFFNESS $C = \beta K$

For $c\omega = h$ = const. at resonance for module r, it is obtained

$$\xi_r = \frac{1}{2}\eta$$

where $\eta = h/k$ is the hysteretic factor or loss factor independent of the excitation pulsation.

For the constant matrix M, the rigidity of the system can be considered as a discrete variable, by the point values for each vibration mode, so that the relationship $\mathbf{C} = \beta\mathbf{K}$ is valid. Experimentally the excitation pulsation ω can be made discreetly variable so that all normal vibration modes can be excited with the pulsations p_r.

It is noted with $\Omega = \omega/p$ and we obtain $\eta = c\omega/k$ where $c = 2\zeta\sqrt{mk}$ or

$$\eta = 2\zeta\frac{\omega}{p} = 2\zeta\Omega$$

and for module r it can be written as follows

$$\zeta_r = \frac{1}{2}\eta\frac{1}{\Omega_r} = \frac{1}{2}\eta\frac{p_r}{\omega} \quad (9)$$

or

$$\zeta_r = A_1 p_r \quad (10)$$

where

$p_r -$ is the pulsation of module r
$\omega -$ exciting pulsation;
$A_1 = \eta/2\omega -$ const.

At resonance $p_r = \omega$ we have $\zeta_r^{rez} = \eta/2$, and outside of the resonances of r order, for each eigenmode p_r the lines presented in Figure 2a, are obtained.

On the other hand, it can be written as:

$$\zeta_r = \frac{1}{2}\eta\frac{1}{\omega}\sqrt{\frac{k_r}{m_r}} \quad (11)$$

or

$$\zeta_r = A_1\sqrt{k_r} \quad (12)$$

represented by the curves in figure 2b, where $A_1 = \eta/2\omega$ is a constant for a given excitation.

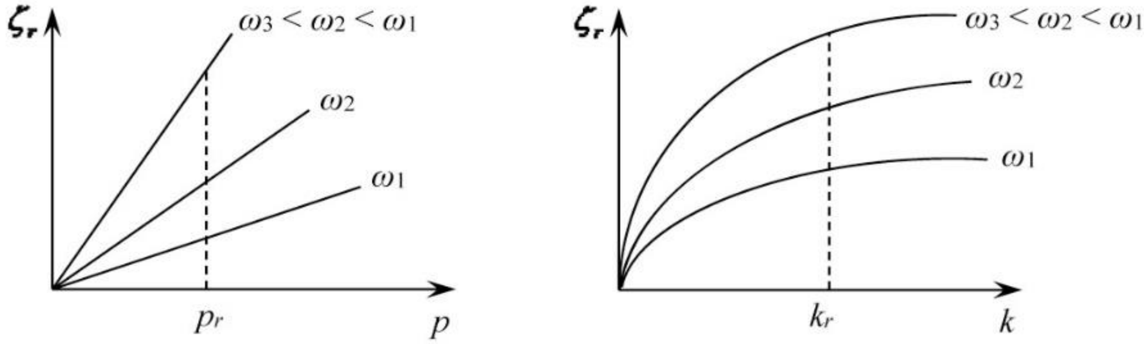

Fig. 2. Definition of resonant frequencies.

3. DAMPING PROPORTIONAL WITH MASS $C = \alpha M$

For the constant matrix $\mathbf{K}$ it is adopted $\mathbf{C} = \alpha\mathbf{M}$ where the damping changes with the discrete values of the inertial elements contained by the matrix M corresponding to the eigenmodes. Thus, the condition $\mathbf{C} = \alpha\mathbf{M}$ for module r can be written as

$$2\zeta_r m_r p_r = \alpha m_r$$

or

$$\alpha = 2\zeta_r p_r = \text{const} \tag{13}$$

We know that

$$\eta = \frac{c\omega}{k} = \frac{\alpha m\omega}{k}$$

from which $\alpha = \eta k/m\omega$, which together with (13) leads to

$$\zeta_r = A_1 p_r \tag{14}$$

Depending on the mass m_r, we have

$$\zeta_r = \frac{\eta}{2\omega}\frac{\sqrt{k_r}}{\sqrt{m_k}}$$

or

$$\zeta_r = A_2\frac{1}{\sqrt{m_k}} \tag{15}$$

where

$$A_2 = \frac{\eta}{2\omega}\sqrt{k_r}.$$

The graphical representation for (14) and (15) is given in Figure 3.

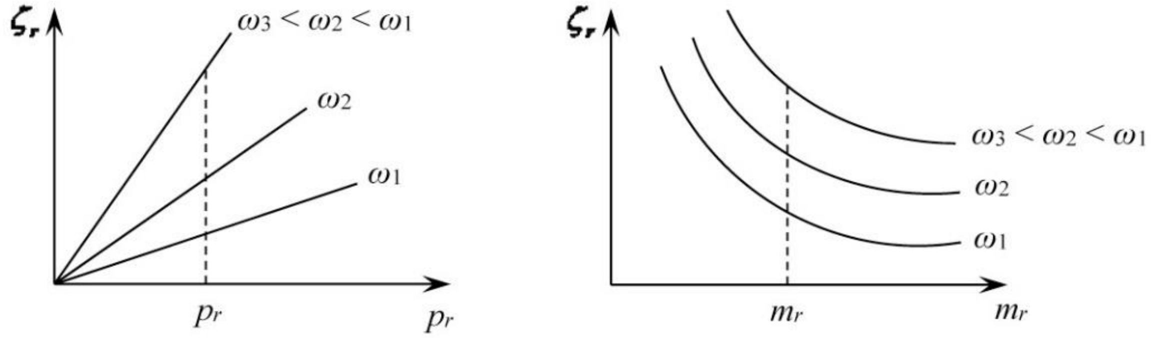

Fig. 3. Graphycal representation of damping coefficients.

4. CONCLUSIONS

The hysteretic elastomer is characterized by the damping which can be expressed as follows:
a) the multiplication $c\omega = h$ is constant which implies that $\eta = c\omega/k$ is constant;
b) the equivalent viscous damping expressed by ζ_r it is a linear function of the system's eigenmodes;
c) hysteretic damping is independent of the excitation pulsation.

Experiments on the specialized dynamic stands have highlighted the legitimacies previously highlighted.

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