Romanian Journal of Mechanics
Volume 1, Issue 1/2016, pp 45-50, ISSN 2537 - 5229, rjm.journals.srmta.ro

STUDIES OF ELASTIC DEFORMATIONS IN LEONARDO'S MANUSCRIPTS

Andrei C. VASILESCU
Technical University of Civil Engineering Bucharest, Romania
E-mail: andrei.vasilescu@utcb.ro

Abstract. Leonardo da Vinci (1452–1519) was the main figure of the Renaissance by his works in many fields of arts and sciences. It was in the same time mathematician, painter, architect and engineer. But his remarkable contributions were also in botany, music, philosophy, anatomy and physiology. Leonardo is revered for his technological ingenuity. "***All our knowledge has its origins in our perceptions***" he noted. Based on the natural ability to observe and understand the details of a phenomenon, he developed his power of attaining the gist in many fields of human knowledge. Briefly, this was a skill named intuition doubled by the gift of quickness and accurate talent to make drawings and sketches. His books of notes and manuscripts are the source of study for scientist, revealing continuously surprises. The paper presents some aspects of the Leonardo's contribution in the domain of "*strength of materials*" referring to the behavior of elastic bars. From the geometrical constructions he studied the qualitative aspects of beam bending, preparing to try the establishment of stresses causing the arch failure. Through experimental studies, Leonardo observed and annotated the phenomenon of elastic deformation of elastic springs. Assuming that cross-section is rectangular, Leonardo concluded that plane sections remain plane and perpendicular to the midplane after deformation. This is the geometrical hypothesis of the Bernoulli-Euler beam theory, presented by Leonard approximately 250 years earlier.
Key words: elastic beam, arches deformations, beam theory

1. INTRODUCTION

Leonardo's notes on Mechanics are extraordinarily numerous. One of the most important sources for historians in mechanics remained the "***Leonardo da Vinci's Notebook Collection***" published by **Jean Paul Richter** [1] in 1883 [1]. Richter organized the text by the titles or headings, which Leonardo himself prefixed to most of these notes. We can quote from the introduction of this chapter XIII (the second volume) entitle "*Theoretical writings on Architecture*":

Fig. 1. Leonardo da Vinci (1452-1519)

> *They might all be collected under the one title: "Studies on the Strength of Materials". Among them the investigations on the subject of fissures in walls are particularly thorough, and very fully reported; these passages are also especially interesting*

[1] **Richter**, **Jean Paul** (1847-1937) was a dealer and historian of Italian art; documentary scholar on Leonardo. In 1883 Richter issued his magnum opus "*Literary Works of Leonardo*" which constituted a re-examination and scholarly translation in English of Leonardo's writings (the contents of Leonardo's notebooks).

because Leonardo was certainly the first writer on architecture who ever treated the subject at all. Here, as in all other cases Leonardo carefully avoids all abstract argument. His data are not derived from the principles of algebra, but from the laws of mechanics, and his method throughout is strictly experimental.

The translations of the Leonardo's notes by Richter, are massively quoted by all vincian writers.

Relatively recently discovered manuscripts known as "Codices Madrid"[1], represent another invaluable source of research in the history of mechanics, the all-encompassing nature of Leonardo da Vinci's intuition is brought to life before our eyes. The Codex Madrid I[2] have a close correspondence with the Atlantic Codex[3], from the "Biblioteca Ambrosiana" in Milan: a series of items (scales, weights, gears, springs, machinery watches, screws, hinges, etc.) are outlined in the Codex Atlanticus and developed in the Codex Madrid I, with drawings with great detail and quality.

2. STUDIES ON ELASTIC DEFORMATIONS

On the first page of Codex Madrid I, Leonardo noted (between paragraphs) a motto: *"Chi nega la ragion delle cose publica la sua ignoranza"*, which can be translated *"Those who deny the reason of things he publishes his ignorance"* (Fig. 2). No comment!

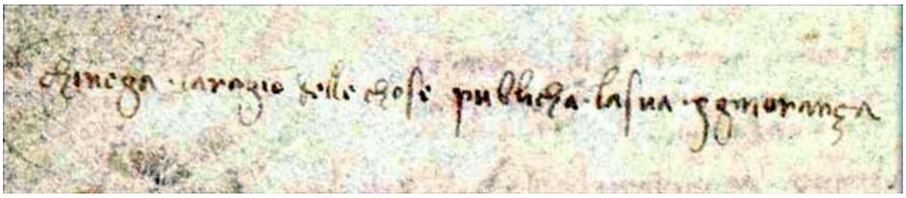

Fig. 2. Excerpt of reverted image (Codex Madrid I) [3]

In the age, the arches were the structural elements most used to cover large openings of churches, palaces, bridges, buildings, etc. From the beginning, we mention that external actions were modeled by forces of gravity, transmitted through wires and pulleys (Fig. 3). The Leonardo's notes surrounding the two figures show the values of the forces in each wire [2].

Figure 3. Codex-Madrid-I, folio 85 verso (detail)

[1] The **Madrid Codices** are two manuscripts by Leonardo da Vinci having been officially lost since 1838, which were discovered in the "*Biblioteca Nacional de España*" in Madrid, in 1967.

[2] **Codex Madrid I** (Mss. 8937), is basically a treatise on mechanics, consists of 12 booklets of 16 pages each.

[3] The **Codex Atlanticus** (Atlantic Codex) comprises 1,119 leaves dating from 1478 to 1519, dealing with various subjects from mechanics to hydraulics, from flight to weaponry or to musical instruments, and from mathematics to botany.

With a painstaking exploration of reality the elastic deformations of structures are represented (Fig. 4 and 5). For instance, we can observe in Figure 5: the first wooden arch is loaded by 3 weights of 4, 8 and 16; the second drawing shows us three cantilevers fixed in a wall, loaded with 16, 8 and 4; the third *ends juxtaposed with tie rod* 4-4; the fourth: *four wooden beams treated with tie rods and weights* 4-1-2-3-4; … ; the eighth: *arch constrained by corner*; the ninth *weight that bends the two arches together* and the tenth: "*eight and pulley, loaded* with 4. In conclusion, Leonardo sketched out the deformations of some models of elastic wooden beams, in different positions and combinations. The loads are specified for some of his drawings (Fig. 4, 5 and 7).

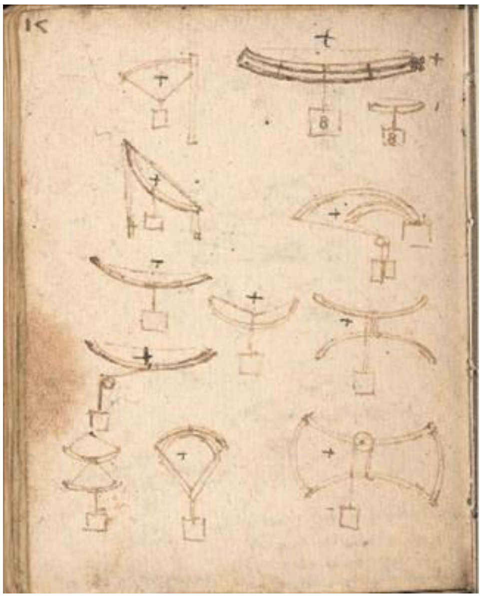

Fig. 4. Twelve figures of curved beams with weights of 8 or 4 lb. (Codex Forster II, f. 88 v.)

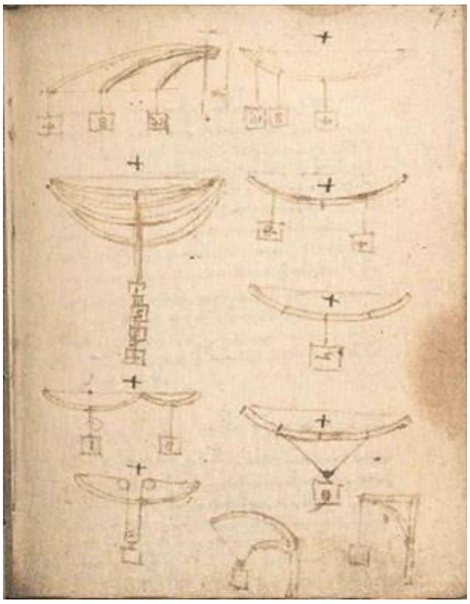

Fig. 5. Ten figures of wooden arches in curved positions. (Codex-Forster II, f. 89 r.)

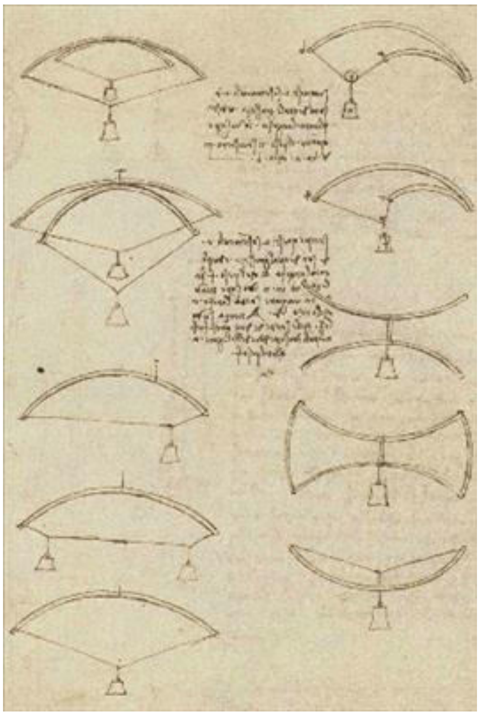

Fig. 6. Ten figures (Codex-Madrid I, f. 135 v.)

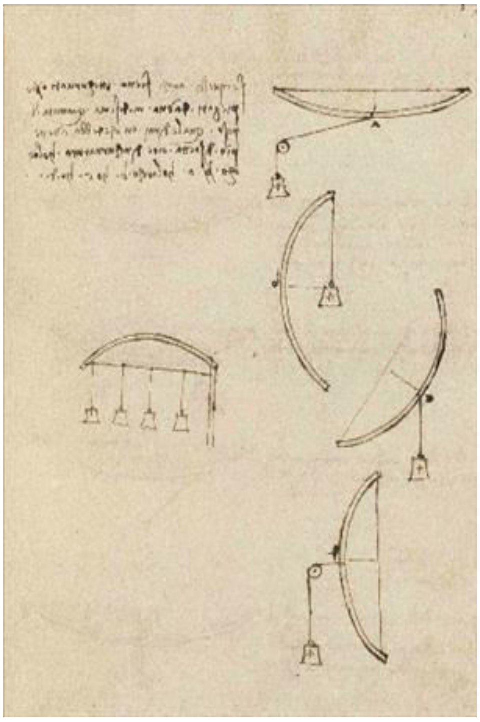

Fig. 7. Five figures (Codex-Madrid I, f. 136 r.)

Figures 6 and 7 show us more carefully drawings and interesting comments. The translated text from Figure 6 reveals us the enunciation of the problem and the question (referring to the drawing from upper right corner): *If these two elastic supports (arches)* ***a*** *and* ***b*** *are of equal thickness and length doubled, I want to know how much of weight* ***n*** *will be applied more in* ***a*** *than in* ***b.***

The text from the center of the page (in Fig. 6) presents a more complicate the problem (referring to the drawing on the right side, the second row): *If these two elastic supports* (arches) ***c*** *and* ***d*** *are of equal thickness and length doubled, and a load* ***f*** *is linked to* ***n****, I want to know how much to increase the arrow of the arch* ***c*** *to* ***d****. However, in the case where I want equal arrows, I want to know where I should be linked* ***f*** *to* ***n****.*

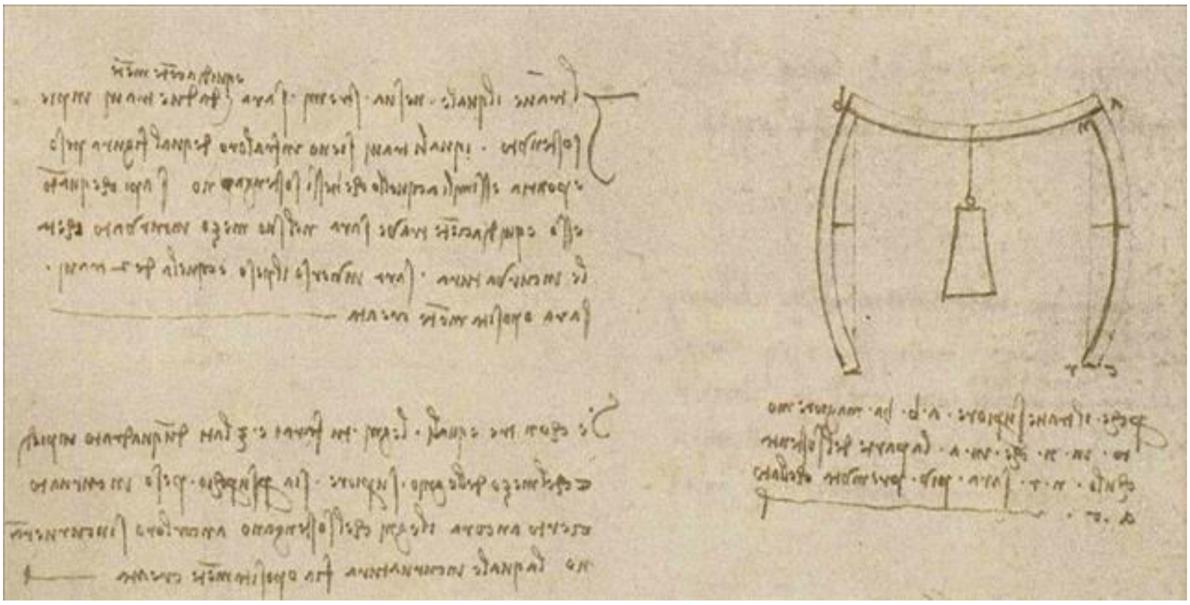

Fig. 8. Codex-MADRID I, folio 139 r.

Under figure of the deformed frame to the right(fig. 8), we can find the transcript of Leonardo note:

> "*Il trave il quale ne' sua estremi sarà equidiacentemente da due travi in piè sostenuto, i quali travi sieno infra loro d'equal figura, peso e potentia, e simili a quello che essi sostengano, sapi che quando esso equidiacente trave sarà nel suo mezo incurvato, che tale incurvatura sarà inverso il peso e quela de' 2 travi sarà opositamente creata.*" [2]

This text can be interpreted like:

> "*The beam which will be supported by two beams to its extremes, which beams are of equal shape, weight and solidity, similar to what they support that when this beam will be loaded at the center and curved that will reverse the sagging of the two beams in opposite position.*"

From the drawing and the note, it is clear that the symmetrical frame loaded in his center, will have a symmetrical sagging and the right angle of the two rigid nodes will remain unchanged after deformation!

3. THE EMERGENT BEAM THEORY FROM LEONARDO DA VINCI

The first analytical attempt at developing the beam theory are thanks to Galileo [4], but recent studies argue that Leonardo da Vinci was the first to make the crucial observations 100 years before Galileo, to what com6monly is referred to as Euler-Bernoulli beam theory [5]. Timoshenko's *History of Strength of Materials* [5] summarized the individual contributions made to beam theory by Galileo, Mariotte, (Jacob) Bernoulli, Euler, Parent, and Saint-Venant. However, Leonardo established all of the essential features of the strain distribution in a beam while pondering the deformation of springs for the specific case of a rectangular cross-section (Figures 8-12). He assumed that plane sections remain plane and perpendicular to the midplane after deformation (this is the Bernoulli-Euler hypothesis - enounced approximately in 1750).

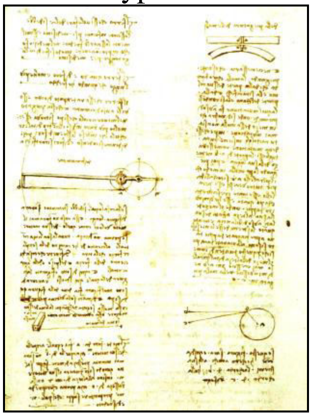

Figure 9. Codex-Madrid I, folio 84 v.

Figure 10. Deformation of a beam/spring with rectangular cross-section (detail - folio 84 v.)

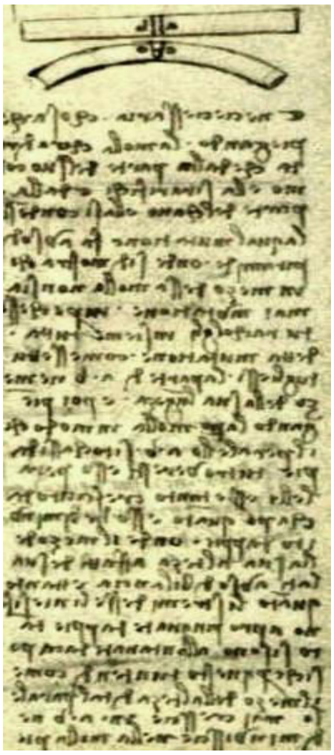

Fig. 11. Codex-Madrid I, folio 84 v. (right side detail)

Of bending of the springs

If a straight spring is bent, it is necessary that its convex part become thinner and its concave part, thicker. This modification is pyramidal, and consequently, there will never be a change in the middle of the spring. You shall discover, if you consider all of the aforementioned modifications, that by taking part '*ab*' in the middle of its length and then bending the spring in a way that the two parallel lines, '*a*' and '*b*' touch a the bottom, the distance between the parallel lines has grown as much at the top as it has diminished at the bottom. Therefore, the center of its height has become much like a balance for the sides. And the ends of those lines draw as close at the bottom as much as they draw away at the top. From this you will understand why the center of the height of the parallels never increases in '*ab*' nor diminishes in the bent spring at '*co*.'

Fig. 12. English Translation of the Leonardo's comment from Fig. 11 [5].

In the text of the Leonardo's note (Figure 7) we can recognize that bending deformation essentially implies shortening and lengthening of "fibers" in the cross-section:

- fibers on the tension side elongate, while fibers on the compression side shorten;
- the length of the middle of the bar remains unchanged;
- the lengthening or shortening of the fibers is proportional to their distance from the middle axis of the bar **[5].**

4. CONCLUSIONS

Leonardo devoted most of his life to understanding nature. His acute perceptions gave him the possibility to understand immediately the essence of the phenomena studied. He used experimentation and careful observation to master drawing and sketches to make scientific observations. His notebooks combine detailed observation with notes of experiments, describing what could be tried. The questions are written near the sketches are testimonies of these remarks and strengthen them!

Referring to the “strengths of materials” annotations, this paper relieved some of his insights, anticipating scientific research by many centuries. The Hooke's law and calculus was unknown for Leonardo da Vinci. Therefore, the mathematical formulation had to wait until the time of Leonhard Euler (1707-1783) and Daniel Bernoulli (1700-1782).

5. ACKNOWLEDGEMENT

I am particularly grateful for the assistance given by Adrian NAE, museum curator and librarian from Brâncoveanu Monastery (Făgăraş, Romania), who provided useful support in documenting this article.

REFERENCES

1. RICHTER, J. P., *The Literary Works of Leonardo da Vinci*., vol. II, London: S. Low, Marston, Searle & Rivington, 1883.
2. BIBLIOTECA LEOARDINA, e-Leo, Archivio digitale di storia della tecnica e della scienza, http://www.leonardodigitale.com/ (accessed on 6 November, 2016)
3. BIBLIOTECA REALE DI MADRID, Leonardo Interactivo Códice Madrid 1, http://www.bne.es/en/Colecciones/Manuscritos/Leonardo/ (accessed on 6 November, 2016)
4. GERE, J.M., TIMOSHENKO S.P., *Mechanics of Materials*, Fourth Edition, PWS Publishing Company, 1997.
5. ZAMMATTIO C., MARINONI A., BRIZIO A. M., *Leonardo the Scientist*, New York : McGraw-Hill, 1980.
6. TIMOSHENKO. S.P., *History of Strength of Materials* (Dover Publications, Inc., New York, 1983.

Received: November 6, 2016