



## OPTIMIZATION OF A PACKING MANIPULATION ROBOT

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**Abstract** The process of folding cartons by the packing manipulation robot is seen as a succession of folds with respect to the position and orientation of a distinct configuration to another. The folds have different geometries of crease lines with different dimensions and profiles. The folding carton into successive distinct configurations is determined by the kinematic geometry theory. This paper deals with optimisation of the packing manipulation process with respect to a minimum execution cycle-time of the succession of configurations of the folded carton and a minimum discord between predictions and measured observations. The concept of line vectors and screw theory associated with the graph technique allows a good describing of the dynamics of carton folding.

*Key words:* kinematics, mobility, configuration, manipulation, screw theory, packaging.

### 1. INTRODUCTION

Origami is the art of folding papers into various shapes, which is often associated with Japanese culture [1]. It is widely applied in the design of carton packaging because of its reconfiguration. The basic feature of the packaging machine is reconfigurability. The machine is equipped with sensors, logic controllers and computers that allow the machine to automatically mount the carton and fill it with a specific product [2].

Mathematical modeling of folding is based on the graph and screw theories, and representing cartons as spatial mechanisms [3]. Dai and Rees Jones [4-6] explore the carton folding as an equivalent mechanism, called a metamorphic mechanism, by taking creases as revolute joints and panels as links. Origami-inspired novel mechanisms were developed including the carton manipulation analysis by using configuration transformation [7] or the planning using dual robotic fingers [8]. An advanced kinematic simulation of a complex carton was modeled in [9] for an adaptable and reconfigurable articulated palm.

The carton is modeled as an equivalent mechanism associated to the folding technique [10, 11] by using the creases represented by revolute hinges and carton panels by links. A crease is a hinge joint between two panels and permits the control of a relative motion of the joined panels and the positional and distinct contortions into which the carton can fold. When the carton is considered as an equivalent mechanism, the crease becomes coincident with the axis of the relative motion. The axes are referred to the instantaneous screw axis (ISA) [12].

Creases are straight lines with edges which may be profiled. The edges and creases with respect to a coordinate system at a panel, fix the shape and size of the panel.

According to our science, no article treats the optimization of folding cartons process into the getting a final box in terms of functionality criteria, minimum steps or minimum packaging time. Packaging machines get boxes by using a fixed strategy with fix steps that are often longer and more cumbersome than necessary.

This paper is suggested by the lack of optimization criteria in the carton packaging technique, and it is concerned with the optimisation of the process in terms of minimum time of execution of configurations of foldable carton, and a minimum discord between predictions and measured observations. The scope of minimisation is to obtain a versatile packaging process adapted to different cartons and final shapes.

A robotic system composed by four fingers, two with three degrees of freedom, and two with two degrees of freedom each, is considered. The robot is presented in Fig.1 [9]. The three-degree-of-freedom fingers provide a yaw (Y) motion at the base and pitch (P) motions on the following two joints forming a Y-P-P configuration.

The two-degree-of-freedom fingers have only pitch motions allowing it to move on a planar surface. These fingers are mounted on linear tracks that can move orthogonally and can be oriented at any angle at their base. Two horizontal jaws shown in the figure are arranged with the pushing direction parallel to the fingers' horizontal tracks. The plates attached to the jaws and are to be mounted on passive joints that are under-actuated i.e. during the pushing operation, they can orient along the shape of the cardboard panels.

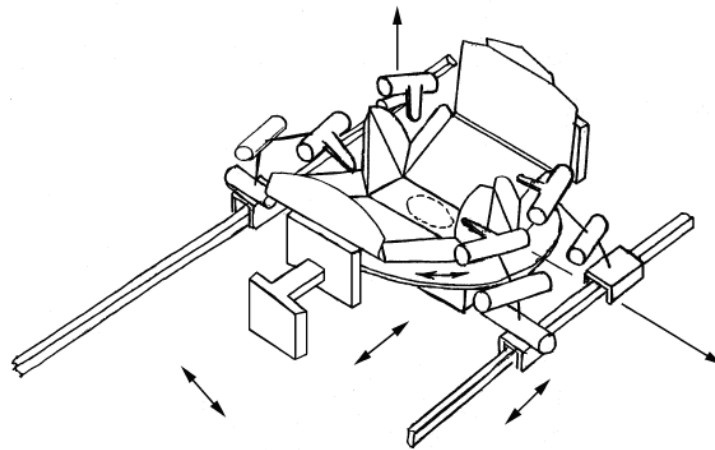


Fig.1. The packaging machine [9].

The optimisation criterion is expressed in term of a minimum cycle-time for execution of the succession of folding configurations and a least discord between predictions and measured observation.

## 2. THE PACKING MANIPULATION ROBOT

The kinematic model of the carton has been studied by Dubey et al.[9-11], with emphases to the equations which describe the folding process of the carton based on the shape transformations. The folding of the carton is achieved by a packaging robot, so each movable element of the machine needs to be connected kinematically. This requires to obtain the solution of the inverse kinematics of the fingers.

The 2-DOF planar finger has the available standard solutions available [15]. The basic angles  $\beta$  and  $\gamma$  are defined as [16] (Fig.2)

$$\beta = \cos^{-1} \left( \frac{R_1 \cos \phi - R_3 \sin \phi}{\cos \alpha} \right), \quad (1)$$

$$\gamma = \cos^{-1} \left( \frac{(R_1 \sin \phi + R_3 \cos \phi) \cos \alpha \sin \beta + R_2 \sin \alpha}{\sin^2 \alpha + (\cos \alpha \sin \beta)^2} \right), \quad (2)$$

where

$$R_1 = (\cos^2 \delta (1 - \cos \theta) + \cos \theta) \cos \alpha - \cos \delta \sin \delta (1 - \cos \theta) \sin \alpha, \quad (3)$$

$$R_2 = (\sin^2 \delta (1 - \cos \theta) + \cos \theta) \sin \alpha - \cos \delta \sin \delta (1 - \cos \theta) \cos \alpha, \quad (4)$$

$$R_3 = \sin \delta \sin \theta \cos \alpha + \cos \delta \sin \theta \sin \alpha, \quad (5)$$

The solution for the 3-dof fingers with yaw-pitch-pitch (Y-P-P) configuration can be expressed in a closed form

$$\theta_1 = \tan^{-1} \left( \frac{p_y}{p_x} \right), \quad (6)$$

$$\theta_3 = \cos^{-1} \left( \frac{p_x^2 + p_y^2 + p_z^2 + l_1^2 (p_x \cos \phi_1 + p_y \sin \phi) - l_2^2 - l_3^2}{2l_2 l_3} \right), \quad (7)$$

$$\theta_2 = \sin^{-1} \left( \frac{(\cos \phi l_2 + l_3 \cos \phi \cos \delta) p_z - (p_x - l_1 \cos \phi) \sin \delta l_3}{(l_2 + l_3 \cos \delta)(\cos \phi l_2 + l_3 \cos \phi \cos \delta) + l_3^2 \cos \phi \sin^2 \delta} \right), \quad (8)$$

where  $p_x, p_y, p_z$  are the coordinates of the target point,  $l_1, l_2, l_3$  are the link lengths of the finger corresponding to joint angles  $\phi, \alpha, \delta$  moving from base to the tip.

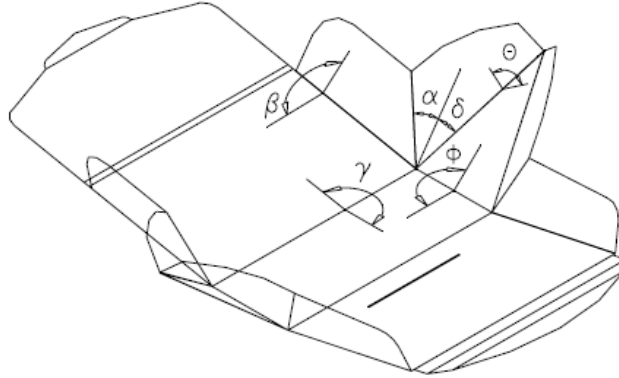


Fig. 2. Kinematic model of the carton [16].

The kinematic connectivity between carton and the fingers can be achieved by locating various contact points on the carton and recording the displacement of these points as the folding of the carton takes place. The carton contact points can be identified by geometrical interpretation of the folding sequences [17]. These contact points are then used to find joint displacement for each finger's joint. The displacement data are further interpolated to generate each finger path.

### 3. SCREW REPRESENTATION OF CARTON

The motion of a carton in space can be reduced to a combination of a rotation and a translation along a line [5, 6, 14]. The motions can be seen as a motion of a screw in a nut, in which any rotation of the screw in the nut is accompanied by a translation, determined by the pitch of the thread, along the axis of the nut.

A line vector is a vector with 6 elements defined as (Fig. 3)

$$L = \{l, r \times l\}, \quad (9)$$

The carton motion can be expressed as a resultant angular velocity  $\omega$  about an axis and a velocity of a point in the body [18]. The vector  $\omega$  gives both the magnitude of the angular velocity and the direction along the instantaneous screw axis (ISA).

The velocity has two components, i.e. one due to the angular velocity  $r \times \omega$ , and the other due to the translation along the axis  $h\omega$ , with a scalar  $h$  representing the pitch [19]. The term  $r \times \omega$  determines the position relative to the origin (the point coincident with the point whose velocity is being cited) and the  $h\omega$  term which determines the translation along the instantaneous screw axis (ISA). This motion, represented as a vector  $T_w$ , is a twist (Fig. 4)

$$T_w = \{\omega, r \times \omega + h\omega\}, \quad (10)$$

or

$$T_w = \omega \{s, r \times s + hs\} = \omega \{s, s_0\}. \quad (11)$$

The screw is a unit vector  $s$  in the ISA direction embedded with its location vector  $r$ , and with an attached pitch, given as

$$S = \{s, r \times s + hs\} = \{s, s_0\}. \quad (12)$$

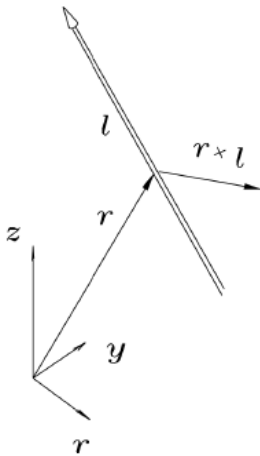


Fig. 3. The line vector.

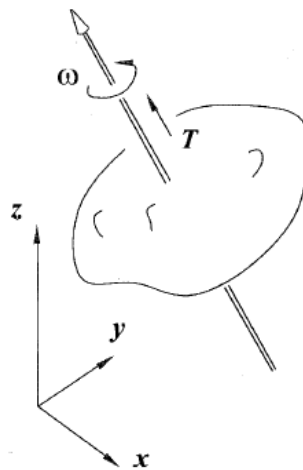


Fig. 4. A twist in the space.

So, a line vector is a screw with a zero pitch which is represented when the dot product in the above equation is identically zero.

An initial carton and the cuboidal box as the final shape is considered, with dimensions depth  $b$ , breadth  $a$  and height  $e$ . Fig. 5 presents the initial configuration of carton blank and its final folded configuration. An example for folding sequence is presented in Fig.6. Fold creases are indicated by dotted lines.

The angles  $\theta$  and  $\varphi$  describe the carton manipulation during the motion. The lines  $c_i, i=1,2,\dots,6$ , are expressed as

$$c_1 = [0 \ 0 \ 1 \ 0 \ 0 \ 0]^T, \quad (13)$$

$$c_2 = [0 \ 0 \ 1 \ 0 \ -a \ 0]^T, \quad (14)$$

$$c_3 = [0 \ 0 \ 1 \ b \sin \theta \ -a - b \cos \theta \ 0]^T, \quad (15)$$

$$c_4 = [0 \ 0 \ 1 \ b \sin \theta \ -b \cos \theta \ 0]^T, \quad (16)$$

$$c_5 = [1 \ 0 \ 0 \ 0 \ e \ 0]^T, \quad (17)$$

$$c_6 = [1 \ 0 \ 0 \ 0 \ e + b \sin \varphi \ -b \cos \varphi]^T, \quad (18)$$

The screw matrix is defined as six screws shown as columns

$$S = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & b \sin \theta & b \sin \theta & 0 & 0 \\ 0 & -a & -a - b \sin \theta & -b \cos \theta & e & e + b \sin \varphi \\ 0 & 0 & 0 & 0 & 0 & -b \cos \varphi \end{bmatrix}. \quad (19)$$

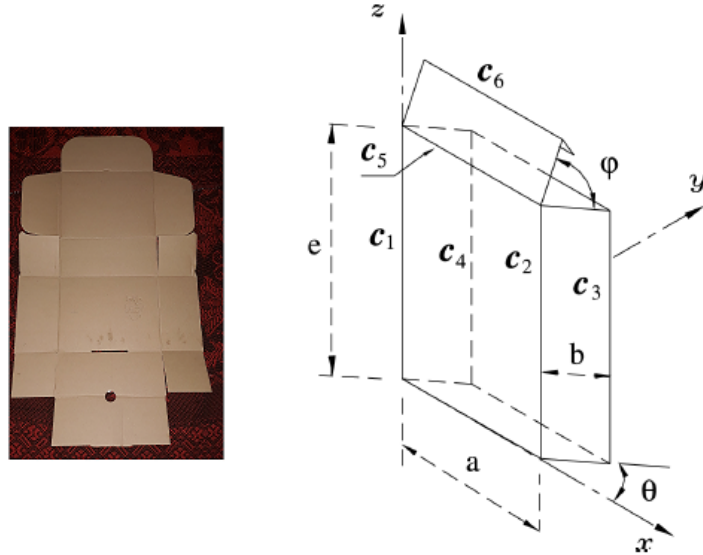


Fig. 5. Initial configuration of carton blank and its folded configuration.

The graph representation of the carton section is represented in Fig. 7.

Mobility is the number of degrees of freedom of a mechanism. A general mobility criterion, is the Kutzbach–Grubler criterion [19]

$$m = d(n - j - 1) + \sum f_i, \quad (20)$$

where  $n$  is the number of links,  $j$  the number of joints and  $f_i$  the number of degrees of freedom of the  $i$ th joint. The coefficient  $d$  presents the order of the screw system of a kinematic chain.

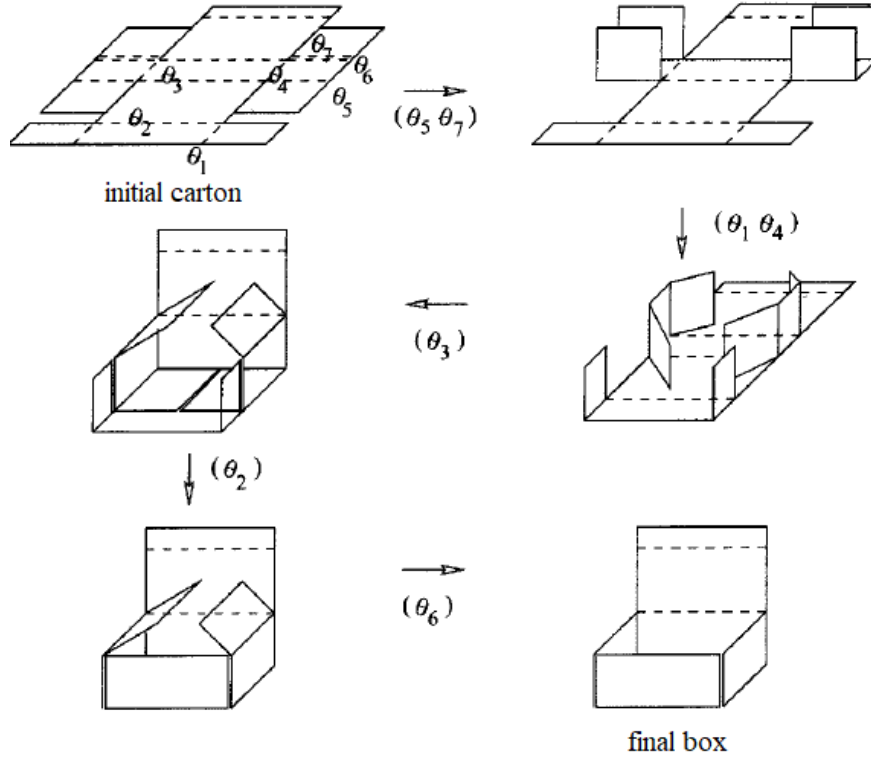


Fig. 6. An example for folding sequence.

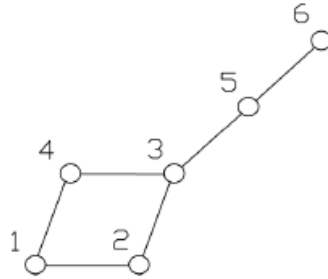


Fig. 7. The graph representation of a carton section.

Since only revolute joints are involved in a carton equivalent mechanism, the number of degrees of freedom  $f_i$  permitted in the  $i$ th joint is 1, i.e.  $\sum f_i = j$ , and (20) becomes

$$m = d(n - j - 1) + j, \quad (21)$$

In our case, for the cuboidal section of the carton with topological graph from Fig. 6, we have  $m = 3(4 - 4 - 1) + 4 + 2(3 - 2 - 1) + 2 = 3$ .

#### 4. Optimization problem

The optimisation problem can be formulated as:

Given the carton shape and its kinematic representation as a robot, find the optimum physically valid paths for each finger between the initial configuration and the final configuration of the folded carton, from the condition of minimum cycle-time for task execution and a minimum discord between predictions and measured observations.

The path or the folding sequence associated for each finger is defined by a joint sequence and a corresponding sequence to take the robot from its unfolded configuration to its folded configuration.

The continuous path for each finger between two target frames is studied in the Cartesian space. The motion path  $s_i$ ,  $i = 1, 2, 3, 4$  of the end-effector of the finger  $i = 1, 2, 3, 4$  characterised by the velocities  $v_i, i = 1, 2, 3, 4$  function of  $s$

$$v_i(s) = \frac{ds_i}{dt}, i = 1, 2, 3, 4. \quad (22)$$

The motion time between two target frames for each finger is

$$t_i = \int_{f_1}^{f_2} \frac{ds}{v(s)}, \quad (23)$$

where  $f_1$  and  $f_2$  are the current frame and the next frame, respectively.

For an uniform motion the time is

$$t = \frac{s}{v}. \quad (24)$$

Let consider that the workspace of the robot is a 3D box. Inside this box, a volumetric grid of points is defined for linking the box to the fingers  $i = 1, 2, 3, 4$ , and the carton, by the trivariate polynomial which defines the deformation function. This can be written in a matrix form

$$X = BP, \quad (25)$$

where  $B$  is the deformation matrix  $ND \times NP$  ( $ND$  is the number of points on the discretized workspace,  $P$  is a matrix  $NP \times 3$  which contains coordinates of the control points and  $X$  is a matrix  $ND \times 3$  with coordinates of the model points at the moment of time  $t$ . The  $NP$  is the number of contact points on the carton considered in order to record the displacement of these points as the folding of the carton takes place. The carton contact points can be identified by geometrical interpretation of the folding sequences. These contact points are then used to find joint displacement for each finger's joint

The box is then *deformed* by the displacement of its lattice, and the position of points of the real fingers is computed.

The displacement field  $\delta X$  between the points of the 3D box and the point data is computed, and then the search for the deformation  $\delta P$  of this box which will best minimize the displacement field  $\delta X$  within a minimum time of execution

$$\text{Minimize}_{t, \delta P} J, J = \|B\delta P(t) - \delta X(t)\|^2. \quad (26)$$

In other words the optimum paths are sought as the minimizer of some distance  $J$  between the measured data and the computed data for a minimum time of execution.

The fitness is written as

$$F = \frac{J_0}{J}, \quad (27)$$

with

$$J_0 = \|B\delta P(t)\|^2. \quad (28)$$

As the convergence criterion of iterative computations, we use the expression  $Z$  to be maximum

$$\text{Maximise } Z, \quad Z = \frac{1}{2} \log_{10} \frac{J_0}{J}. \quad (29)$$

The quality of the model depends on the maximum value of the function  $Z$ .

A genetic algorithm is used to solve (29).

As we said before, the genetic algorithm found the optimum paths for each finger between the initial configuration and the final configuration of the folded carton, from the condition of minimum cycle-time for task execution and a minimum discord between predictions and measured observations.

The genetic parameters are assumed to be as follows: number of populations 200, ratio of reproduction 1.0, number of multi-point crossovers 1, probability of mutation 0.5 and maximum number of generations 500. Our numerical experiments show that for the number of generation less than 100, the genetic algorithm has no solution. For a generation number above this value, we obtain one or two solutions (Fig.8).

The optimal solution has been obtained after 113 iterations (Fig.8). The solution gives the optimal path for all fingers and the minimum time of  $t = 22$  sec, of execution the task of these fingers, are shown in Figs. 9-12.

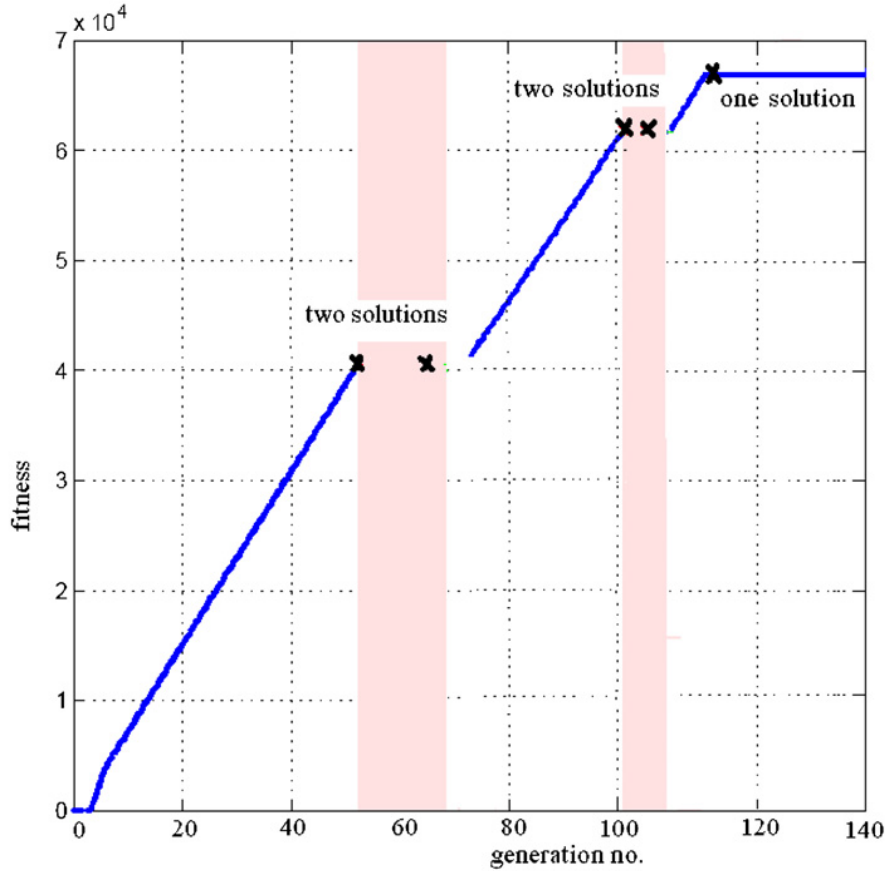


Fig.8. Number of solutions of the genetic algorithm



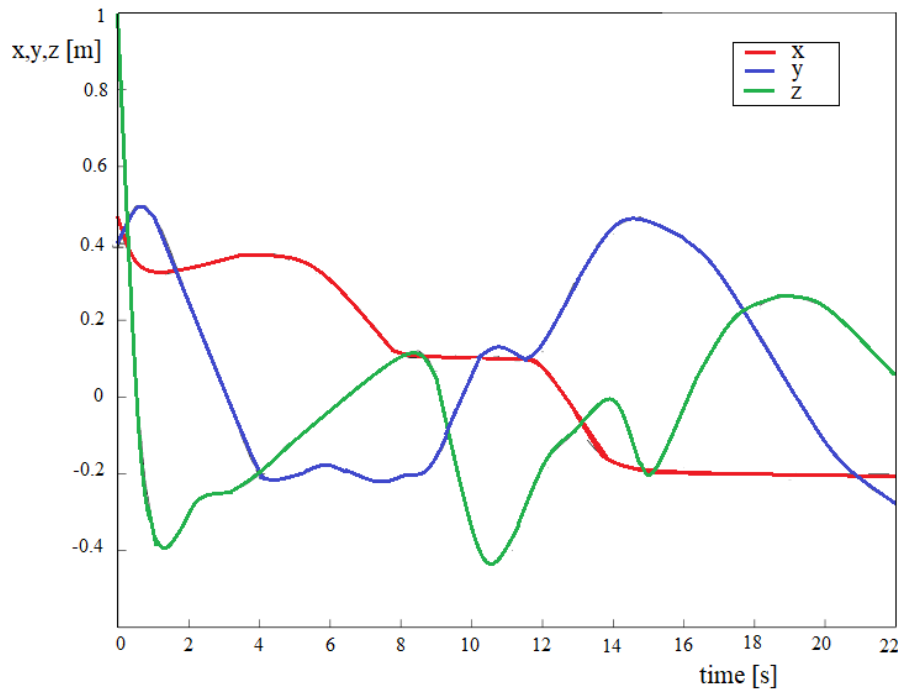


Fig.9.The optimal path for finger 1 with respect to minimum time.

Fig. 9 presents the coordinates  $(X,Y,Z)$  of the optimum path for the finger 1, with respect to the minimum time of execution  $t = 22$  sec. Fig.10 presents the coordinates  $(X,Y,Z)$  of the optimum path for the finger 2, with respect to the same minimum time, while Fig. 11 presents the coordinates  $(X,Y,Z)$  of the optimum path for the finger 2, with respect to the minimum time of execution.

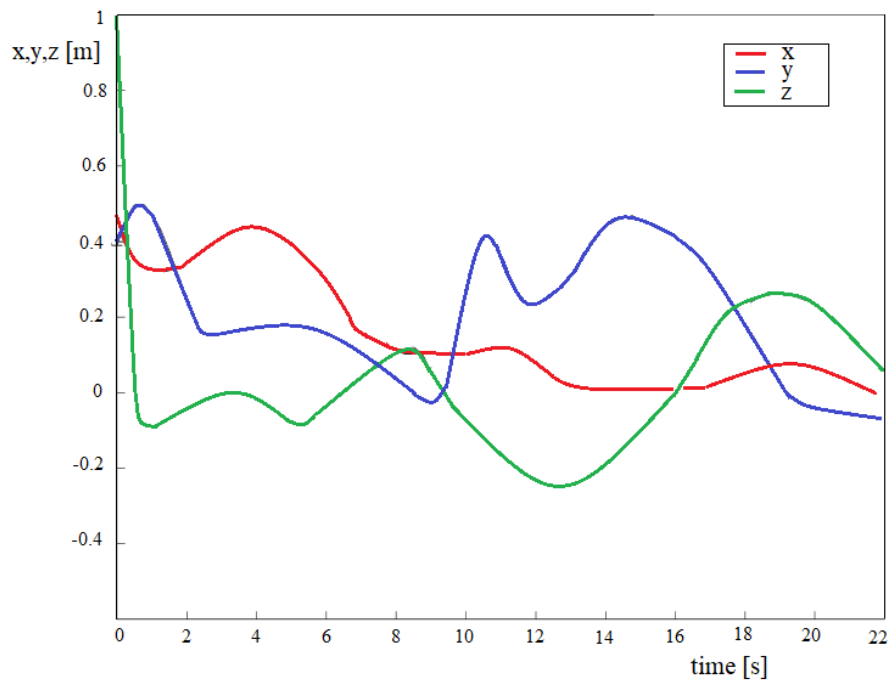


Fig. 10. The optimal path for finger 2 with respect to minimum time.

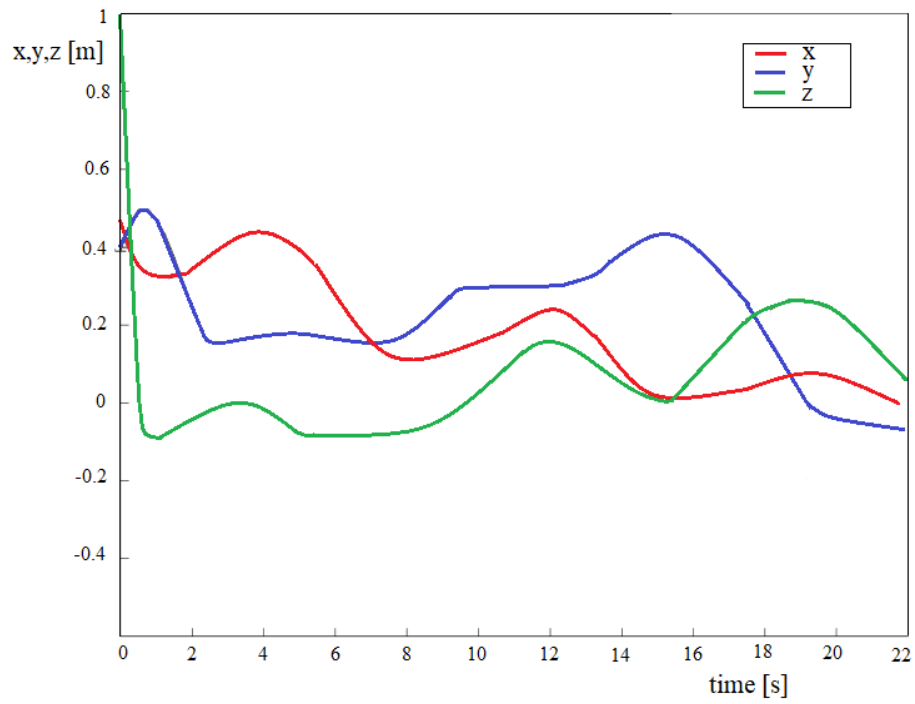


Fig. 11. The optimal path for finger 3 with respect to minimum time.

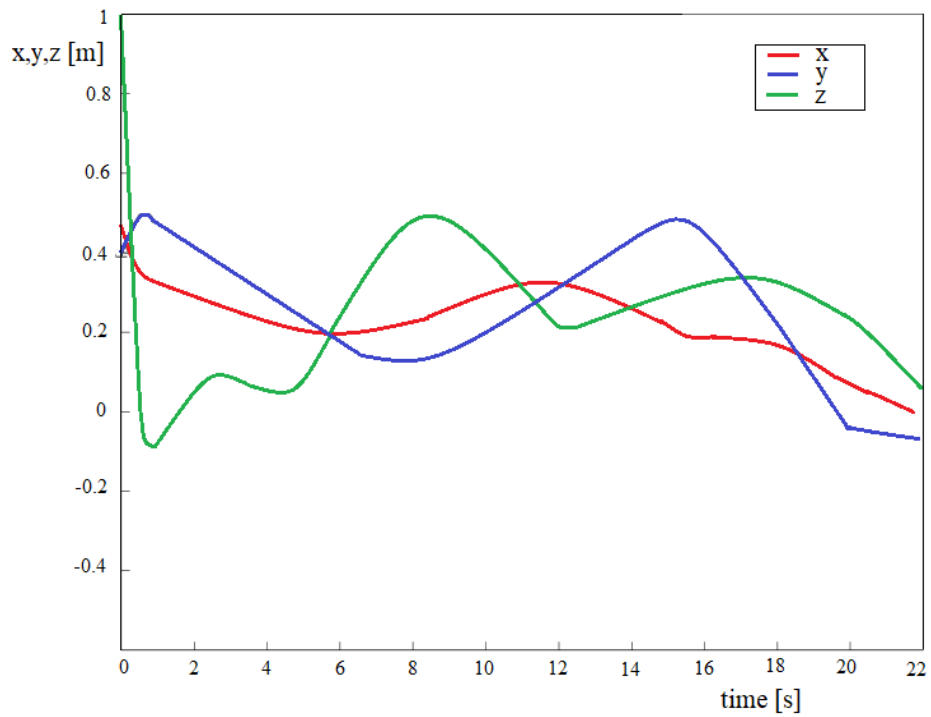


Fig. 12. The optimal path for finger 4 with respect to minimum time.

## 5. CONCLUSIONS

This paper deals with optimisation of the packing manipulation process of a robotic system composed by four fingers, two with three degrees of freedom, and two with two degrees of freedom each. The optimisation approach determines the optimum physical valid paths for all fingers from the condition of minimum cycle-time for task execution and a minimum discord between predictions and measured observations.

The concept of line vectors and screw theory associated with the graph technique allows a good describing of the dynamics of carton folding.

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